

## Frequency Response Analysis (Part - I)

1. A system has fourteen poles and two zeros. Its high frequency asymptote in its magnitude plot having a slope of:

- (a) – 40 dB/decade (b) – 240 dB/decade  
(c) – 280 dB/decade (d) – 320 dB/decade

[GATE 1987: 2 Marks]

**Soln.**

Poles (P) = 14

Zeros (z) = 2

$$P - Z = 14 - 2 = 12$$

$$\lim_{\omega \rightarrow \infty} \text{slope} = (P - Z)(-20 \text{ dB/dec}) \\ = -240 \text{ dB/decade}$$

**Ans: Option (b)**

2. The polar plot of  $(s) = \frac{10}{s(s+1)^2}$  intercepts real axis at  $\omega = \omega_0$ . Then, the real part and  $\omega_0$  are respectively given by:

- (a) – 2.5, 1 (b) – 5, 0.5 (c) – 5, 1 (d) – 5, 2

[GATE 1987: 2 Marks]

**Soln.**

$$(s) = \frac{10}{s(s+1)^2} = \frac{10}{s(s+1)(s+1)}$$

$$\angle(j\omega) = -90^\circ - 2 \tan^{-1} \omega$$

$\omega_{pc}$  is the phase cross over frequency where

$$\angle(j\omega) = -180^\circ$$

$$\text{so, } -180^\circ = -90^\circ - 2 \tan^{-1} \omega_{pc}$$

$$2 \tan^{-1} \omega_{pc} = 90^\circ \Rightarrow \omega_{pc} = \tan 45^\circ$$

$$\omega_{pc} = 1 \text{ rad/sec}$$

$$|G|_{\omega=\omega_{pc}} = \frac{10}{\omega \sqrt{1+\omega^2} \sqrt{1+\omega^2}} \\ = \frac{10}{1 \cdot 2 \cdot 2} = \frac{10}{4} = 5$$

At  $\omega = \omega_{pc}$  the polar plot crosses the negative real axis at – 5

**Ans: Option (c)**

## Frequency Response Analysis (Part - I)

**3. From the Nicholas chart one can determine the following quantities pertaining to a closed loop system:**

- (a) Magnitude and phase  
(b) Band width  
(c) Only magnitude  
(d) only phase

[GATE 1989: 2 Marks]

**Soln.** Nicholas chart is magnitude versus phase plot

**Ans: Option (a)**

**4. The open-loop transfer function of a feedback control system is**

(s).  $H(s) = \frac{1}{(s+1)^3}$  The gain margin of the system is

- (a) 2  
(b) 4  
(c) 8  
(d) 16

[GATE 1991: 2 Marks]

**Soln.**  $G(s) \cdot H(s) = \frac{1}{(s+1)^3}$   
 $GM = \frac{1}{|G(j\omega_{pc}) H(j\omega_{pc})|} = \frac{1}{M}$

$\omega_p$  is the phase cross over frequency where

$$\angle(s)H(s) = -180^\circ$$

$$G(s)H(s) = \frac{1}{(s+1)(s+1)(s+1)}$$

$$-3 \tan^{-1} \omega_{pc} = -180^\circ$$

$$\tan^{-1} \omega_{pc} = 60^\circ \Rightarrow \omega_{pc} = \tan(60^\circ)$$

$$\omega_{pc} = 3 \text{ rad/sec}$$

$$M = |(j\omega_{pc}) H(j\omega_{pc})| = \frac{1}{(1 + \omega_{pc}^2)^3} = \frac{1}{8}$$

$$GM = \frac{1}{M} = 8$$

**Ans: Option (c)**

## Frequency Response Analysis (Part - I)

5. Non-minimum phase transfer function is defined as the transfer function

- (a) which has zero in the right-half s-plane
- (b) which has zero only in the left-half s-plane
- (c) which has poles in the right-half s-plane
- (d) which has poles in the left-half s-plane

[GATE 1995: 1 Mark]

**Soln.** Non minimum phase transfer function is defined as the transfer function which has one or more zeros in the right half of s – plane and remaining poles and zeros in the left half of s – plane.

**Ans: Option (a)**

6. The Nyquist plot of a loop transfer function  $(j\omega)$  of a system encloses the  $(-1, j0)$  point. The gain margin of the system is

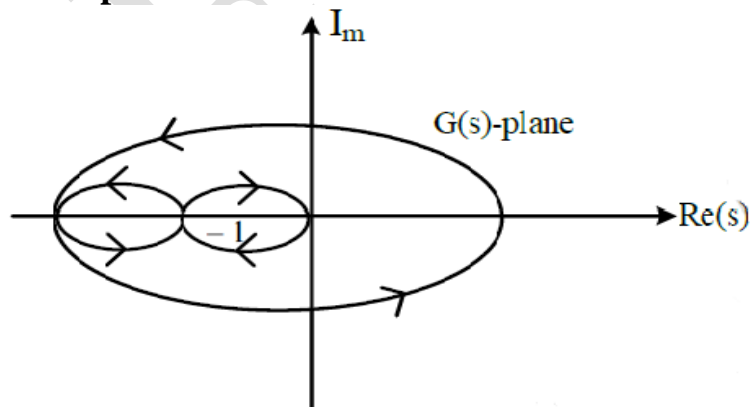
- (a) less than zero
- (b) zero
- (c) greater than zero
- (d) infinity

[GATE 1998: 1 Mark]

**Soln.** A system is unstable when Nyquist plot of  $(j\omega)$  enclosed the point  $(-1, j0)$ . Gain margin of unstable system is less than zero

**Ans: Option (a)**

7. The Nyquist plot for the open-loop transfer function  $G(s)$  of a unity negative feedback system is shown in the figure, if  $G(s)$  has no pole in the right-half of s-plane, the number of roots of the system characteristic equation in the right-half of s-plane is



- (a) 0
- (b) 1
- (c) 2
- (d) 3

[GATE 2001: 1 Mark]

**Soln.**

$$N = P - Z$$

One encirclement in clockwise direction and one in anticlockwise direction house

$$N = 0$$

## Frequency Response Analysis (Part - I)

Given that number of poles of  $(s)G(s)$  in the right half  $s$  – plane,  $p = 0$

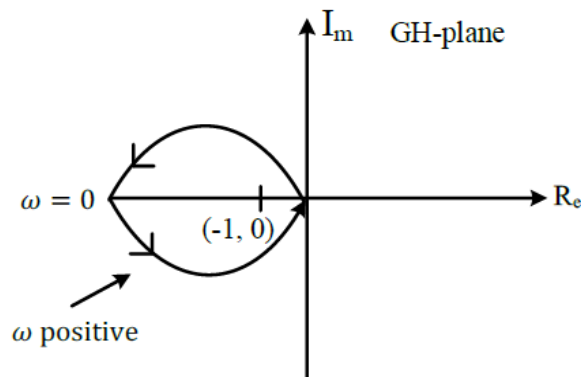
$$N = P - Z$$

$$\text{Or } Z = P - N = 0$$

So No roots of the characteristic equation or poles of the closed loop system lie in RH of  $s$  – plane

**Ans: Option (a)**

**8. In the figure, the Nyquist plot of the open-loop transfer function  $G(s)H(s)$  of a system is shown. If  $G(s)H(s)$  has one right-hand pole, the closed-loop system is**



- (a) always stable
- (b) unstable with one closed-loop right hand pole
- (c) unstable with two closed-loop right hand poles
- (d) unstable with three closed-loop right hand poles

**[GATE 2003: 1 Mark]**

**Soln.**

$$N = P - Z$$

The encirclement of critical point  $(-1, j 0)$  is in the anticlockwise direction hence

$$N = 1, P = 1 \text{ (given)}$$

$$Z = P - N = 0$$

Hence no poles of closed loop system lie in the RH of  $s$  – plane therefore system is always stable.

**Ans: Option (a)**

**9. A system has poles at 0.01 Hz, 1 Hz and 80 Hz; zero at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is**

- (a)  $-90^\circ$
- (b)  $0^\circ$
- (c)  $90^\circ$
- (d)  $-180^\circ$

**[GATE 2004: 2 Marks]**

## Frequency Response Analysis (Part - I)

**Soln.**

Phase shift are

Due to Pole at 0.01 Hz =  $-90^\circ$

Due to Pole at 1 Hz =  $-90^\circ$

Due to Pole at 80 Hz = 0

Not to be considered as the system response at 20 Hz is to be considered

Zero at 5 Hz =  $90^\circ$

Zero at 100 Hz = not be considered

Zero at 200 Hz = not be considered

Thus approximate total phase shift =  $-90 - 90 + 90 = -90^\circ$

**Ans: Option (a)**

**10. The Nyquist plot of  $(j\omega)(j\omega)$  for a closed loop control system, passed through  $(-1, j 0)$  point in GH plane. The gain margin of the system in dB is equal to**

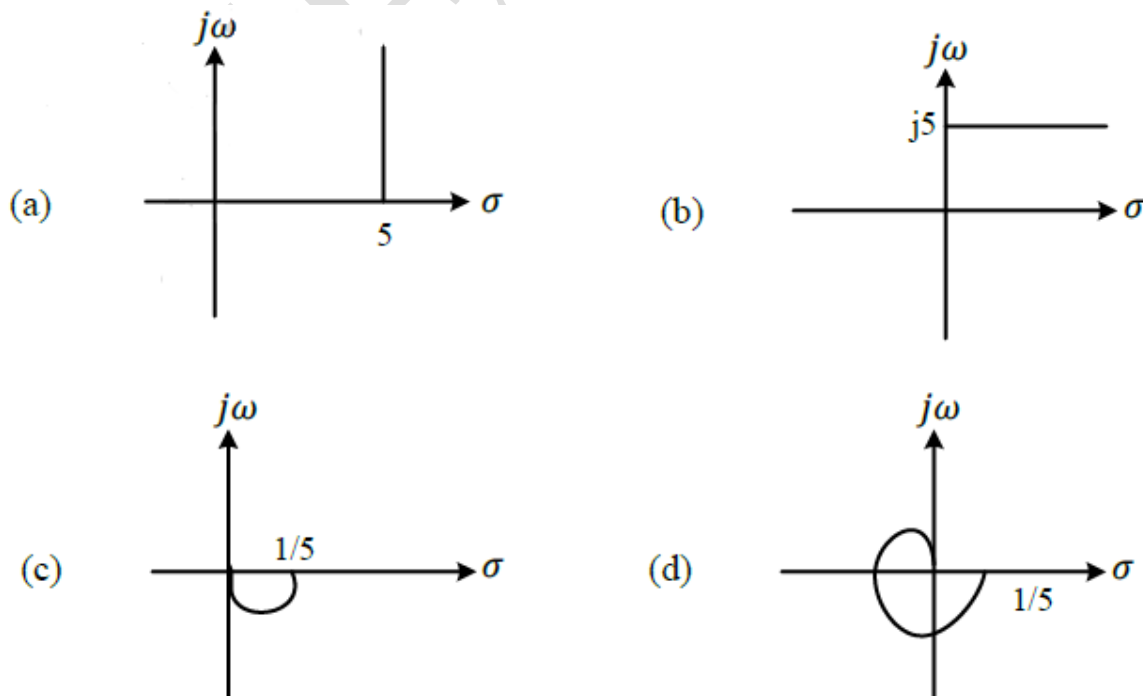
- (a) infinite                      (b) greater than zero                      (c) less than zero                      (d) zero

[GATE 2006: 2 Marks]

**Soln.** The gain margin of system is negative i.e. less than zero

**Ans: Option (c)**

**11. For the transfer function  $(j\omega) = 5 + j\omega$ , the corresponding Nyquist plot for positive frequency has the form**



## Frequency Response Analysis (Part - I)

**Soln.** The transfer function  $(j\omega) = 5 + j\omega$

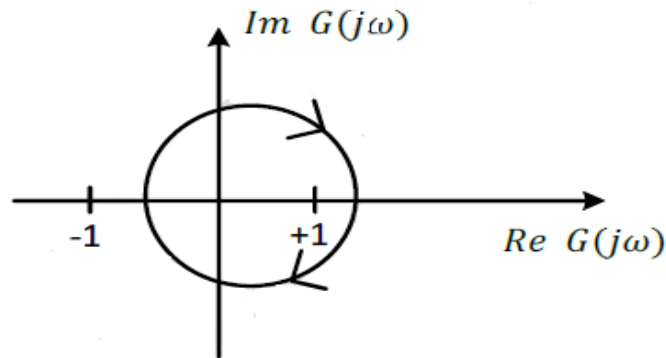
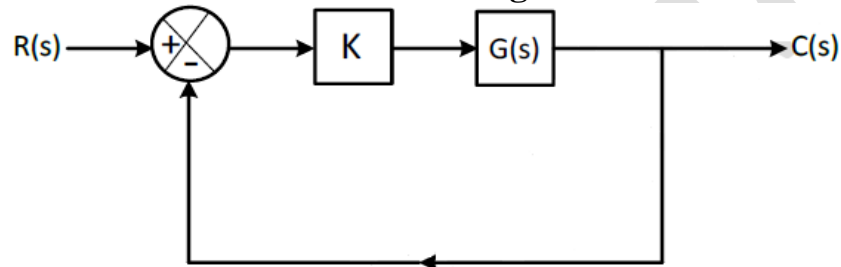
$$|(j\omega)| = \sqrt{25 + \omega^2}$$

At  $\omega = 0$ ,  $|(0)| = 5$

At  $\omega = \infty$ ,  $|(\infty)| = \infty$

**Ans: Option (a)**

**12. Consider the feedback system shown in the figure. The Nyquist plot of  $G(s)$  is also shown. Which one of the following conclusions is correct?**



- (a)  $G(s)$  is an all-pass filter
- (b)  $G(s)$  is strictly proper transfer function
- (c)  $G(s)$  is a stable and minimum-phase transfer function
- (d) The closed-loop system is unstable for sufficiently large and positive  $K$ .

**Soln.** Nyquist plot is not enclosed critical point  $(-1, j 0)$ , hence the system is stable. If the value of gain  $K$  is increased, then intersection point moves towards  $-\infty$  on the negative real axis which makes system unstable.

**Ans: Option (d)**